PHYSICAL REVIEW VOLUME 129, NUMBER 5 1 MARCH 1963

Experimental Consequences of the Majorana Theory for the Muon's Neutrino

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(Received 15 October 1962)

The Majorana theory applied to the muon's neutrino predicts lepton-nonconserving phenomena with a small probability of second order in m_ν, the neutrino mass, and a, the deviation from $V-A$ theory. In particular, in the coupled reactions $\pi \rightarrow \mu+\nu_\mu$, $\nu_\mu+Z \rightarrow Z'+\mu$, where Z and Z' are nuclei, there is a small probability that the muon produced in the second reaction has the same sign as the original π. This probability is computed to lowest order in a and m_ν. It may be as great as 8/100, consistent with present experimental upper limits on a and m_ν. If we assume $a=0$, the probability goes as m_ν^2 and cannot be more than 1/200.

If a small lepton-nonconserving effect were to be observed, the problem might arise of distinguishing between nonvanishing a and nonvanishing m_ν. This might be done by an improved measurement of the high-energy part of the electron spectrum in μ decay. To this end the electron spectrum is calculated exactly as a function of energy, angle, m_ν, and a. It is seen that m_ν has an enhanced influence on the high-energy part, whereas the influence of a is of the same order of magnitude at all energies.

THE Majorana theory of neutral leptons treats a particle ν as indistinguishable from its antiparticle $\bar{\nu}$. If ν is massless and interacts only with a factor $(1+\gamma_5)$, the Majorana theory is identical to the lepton-conserving theory, in which ν and $\bar{\nu}$ are distinct. However, Case[1] has pointed out that if ν has a mass m_ν or if it interacts with a factor $[1+\gamma_5+a(1-\gamma_5)]$, the Majorana theory predicts effects, such as neutrinoless double β decay, which violate the law of lepton conservation. The likelihood of these effects would be proportional to m_ν^2 or a^2.

This consideration is particularly noteworthy for the muon's neutrino, ν_μ, which has been shown to be not identical to the electron's neutrino.[2] For the muon's neutrino, the experimental upper limits to m_ν and $|a|$ are still rather high. Therefore it is conceivably possible to observe phenomena that do not conserve leptons. Consider the coupled reactions

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \nu_\mu + Z \rightarrow Z' + \mu^- \tag{1}$$

and also

$$\pi^- \rightarrow \mu^- + \nu_\mu, \quad \nu_\mu + Z \rightarrow Z'' + \mu^+; \tag{2}$$

$$\pi^- \rightarrow \mu^- + \nu_\mu, \quad \nu_\mu + Z \rightarrow Z' + \mu^-; \tag{3}$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \nu_\mu + Z \rightarrow Z'' + \mu^+. \tag{4}$$

(Z, Z', and Z'' represent nuclear species.) If leptons are conserved, then (3) and (4) are absolutely forbidden, since the net effect is to produce two muons of the same sign. But under the Majorana theory, (3) and (4) are possible unless a and m_ν both vanish. Lepton conservation is then replaced, for μ and ν_μ, by a weaker restriction, that the net change in the total number of μ^+, μ^-, and ν_μ present must be even.

Let $R_1, \cdots, R_4$ be the relative rates of reactions (1) through (4). An experiment could measure the branching ratios R_4/R_1 and R_3/R_2. These ratios cannot be computed theoretically since we do not know the matrix elements for $Z \rightarrow Z'$ and $Z \rightarrow Z''$. However, (1) and (3), which involve the same nuclear transition, can be compared theoretically; so can (2) and (4). The product R_3R_4/R_1R_2 can thus be obtained from both theory and experiment.

Let us now designate the helicity of the neutrino by a subscript R or L. Reactions (1) proceed mainly as follows:

$$\pi^+ \rightarrow \mu^+ + (\nu_\mu)_L, \quad (\nu_\mu)_L + Z \rightarrow Z' + \mu^-. \tag{1a}$$

Reactions (3) proceed in any of three ways:

$$\pi^- \rightarrow \mu^- + (\nu_\mu)_L, \quad (\nu_\mu)_L + Z \rightarrow Z' + \mu^-; \tag{3a}$$

$$\pi^- \rightarrow \mu^- + (\nu_\mu)_R, \quad (\nu_\mu)_L + Z \rightarrow Z' + \mu^-; \tag{3b}$$

$$\pi^- \rightarrow \mu^- + (\nu_\mu)_R, \quad (\nu_\mu)_R + Z \rightarrow Z' + \mu^-. \tag{3c}$$

[In (3b), the neutrino is right handed in the π rest system but left handed in the lab system. This is possible if $m_\nu \neq 0$, since the helicity of a massive particle is not Lorentz-invariant.] Thus we may write

$$R_3 = R_{3a} + R_{3b} + R_{3c}. \tag{5}$$

(In the following formulas we assume that a is real, as required by time-reversal invariance.)

The ratio R_{3a}/R_1 is, to lowest order in a and m_ν, the probability that the neutrino from π-μ decay has the "wrong" helicity:

$$\frac{R_{3a}}{R_1} \approx a^2 + \frac{2m_\pi^2}{m_\mu(m_\pi^2 - m_\mu^2)} m_\nu a + \frac{m_\pi^4}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} m_\nu^2, \tag{6}$$

where m_π, m_μ are the rest masses of π and μ.

The ratio R_{3b}/R_1 is given by

$$R_{3b}/R_1 = (1-\cos\phi)/(1+\cos\phi), \tag{7}$$

where ϕ is the angle, in the neutrino rest system, between the velocities of the lab and the pion. This angle is related to θ, the angle between the ν_μ velocity as seen from the π rest system and the π velocity in

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the lab system, by the formula

$$\sin\phi=\frac{p_\pi}{m_\pi}\sin\theta\left[\frac{(E_\pi E_\nu+p_\pi p_\nu\cos\theta)^2}{m_\pi^2 m_\nu^2}-1\right]^{-1/2}, \quad (8)$$

where E_π, p_π are the energy and momentum of π in the lab system, and E_ν, p_ν are the energy and momentum of ν_μ in the π rest system. If we assume that ϕ is small and that $p_\pi^2 \gg m_\pi^2$, we can write

$$\frac{R_{3b}}{R_1}\approx\tfrac{1}{4}\sin^2\phi\approx\frac{m_\pi^2}{(m_\pi^2-m_\mu^2)^2}m_\nu^2\frac{\sin^2\theta}{(1+\cos\theta)^2}. \quad (9)$$

In practice, ϕ and θ are not fixed sharply but are spread out over a range. Thus (9) must be replaced by

$$\frac{R_{3b}}{R_1}\approx\frac{m_\pi^2}{(m_\pi^2-m_\mu^2)^2}m_\nu^2 s, \quad (10)$$

where s is some weighted average of $\sin^2\theta/(1+\cos\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$, the weighting to be determined by the geometry of the experiment. As an estimate, we may take the average uniformly over the solid angle defined by θ between 0 and some $\theta_{\max}<\pi$. Then

$$s=\frac{2}{1-\cos\theta_{\max}}\ln\frac{2}{1+\cos\theta_{\max}}-1. \quad (11)$$

It is reasonable for $\theta_{\max}$ to be quite large, since in the lab system the ν_μ angular distribution is "folded" forward. If $\theta_{\max}=\pi/2$, (11) gives $s=2\ln 2-1\approx 0.4$.

The ratio R_{3c}/R_1 is obtained by comparing cross sections for the same nuclear reaction produced by a neutrino of "wrong" and "right" helicity. To lowest order it is a polynomial homogeneous of second degree in a and m_ν. It is easily seen that the coefficient of a^2 is 1, regardless of what nuclear process is involved. The coefficients of $m_\nu a$ and m_ν^2, however, depend strongly on the likelihood that the μ helicity is opposite that of the neutrino. This in turn depends on the nuclear matrix elements, which at energies of several hundred MeV are quite unknown. At such high energies, retardation cannot be neglected; the matrix elements are therefore not the same as in β decay, nor can the Fermi or Gamow-Teller selection rules be used. Moreover, several different processes may contribute to the cross section, including nuclear breakup and pion emission. It can be shown, however, that the coefficients of $m_\nu a$ and m_ν^2 in R_{3c}/R_1 are almost certainly smaller than the corresponding coefficients in R_{3a}/R_1 by factors of order of magnitude $m_\pi m_\mu/E_\pi E_\mu$ and m_π^2/E_π^2, respectively. (E_μ is the energy of the muon that comes from the nuclear reaction.) Therefore we neglect these terms and write

$$R_{3c}/R_1\approx a^2. \quad (12)$$

Combining (5), (6), (10), and (12), we have

$$\frac{R_3}{R_1}\approx 2a^2+\frac{2m_\pi^2}{m_\mu(m_\pi^2-m_\mu^2)}m_\nu a+\frac{m_\pi^4}{m_\mu^2(m_\pi^2-m_\mu^2)^2}\left(1+\frac{m_\mu^2}{m_\pi^2}s\right)m_\nu^2, \quad (13)$$

where s is as discussed following (10). Since this formula does not involve the nature of the transition $Z\rightarrow Z'$, it holds equally for R_4/R_2.

If phenomena such as (3) and (4) should be observed in the laboratory, the problem would arise of determining the two unknowns, a and m_ν. Equation (13) gives one relation between the two; a second is needed. For example, if $R_3/R_1=10^{-3}$, it might signify either that $a=0$ and $m_\nu\approx 1.6$ MeV, or that $a\approx 0.022$ and $m_\nu=0$. It is not much help to vary s by changing the geometry, or to vary the material used to capture the neutrinos, since the effect on R_3/R_1 of either of these changes is small and hard to estimate. An entirely different phenomenon must be studied.

Such a phenomenon is μ decay. Consider a positive muon at rest, with mass m_μ and polarization $\mathbf{P}$, decaying into a positive electron and two neutrinos. Let the electron have mass m_e, momentum $\mathbf{p}$, and energy E. Let the electron's neutrino be massless, and the muon's neutrino have mass m_ν; and let the relevant term in the interaction Lagrangian be

$$iG\psi_\mu^\dagger\gamma_4\gamma_\alpha[(1+\gamma_5)+a(1-\gamma_5)]\psi_{\nu_\mu}\psi_{\nu_e}^\dagger \times(1+\gamma_5)\gamma_\alpha\gamma_4\psi_e. \quad (14)$$

The probability per unit time that the electron is emitted into an energy interval dE and solid angle $d\Omega$ is then (we take $\hbar=c=1$)

$$\begin{aligned}RdEd\Omega=&(1/4\pi^4)G^2|\mathbf{p}|m_\mu(1-b)^2\\&\times\{[(1+b+2a^2)E(W_0-E)\\&+\tfrac{1}{3}(1+2b)p^2-am_\nu(E-m_e^2/m_\mu)]\\&+\mathbf{p}\cdot\mathbf{P}[\tfrac{1}{3}(1+2b)(E-m_e^2/m_\mu)\\&-\tfrac{1}{3}(1-b-6a^2)(W_0-E)-am_\nu]\}dEd\Omega, \quad (15)\end{aligned}$$

where $W_0=(m_\mu^2+m_e^2)/2m_\mu$, $b=m_\nu^2/2m_\mu(W_0-E)$. This formula is exact except for radiative correction and holds for either the Majorana or the lepton-conserving theory. On integration over Ω it yields the isotropic spectrum given by Michel.[3] [In his equation (45), set $g_1=g_3=g_5=0$, $g_2=\sqrt{2}G(1+a)$, $g_4=\sqrt{2}G(1-a)$.] For $m_\nu=0$ it yields the asymmetry formula given by Bouchiat and Michel[4] [set $Q=(8/\pi)G^2(1+a^2)$, $\rho=\frac{3}{4}(1+a^2)^{-1}$, $\eta=0$, $\alpha=a^2(1+a^2)^{-1}$, $\beta=\frac{1}{4}(1+a^2)^{-1}$, and replace m_e^2/E by m_e^2/m_μ in the coefficient of β]. For $a=0$ it reduces to the formula of Bahcall and Curtis.[5] [If, in their Eq. (10), G^2 is the same as ours, then W_0 should be replaced by m_μ in the definition of A.]

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Since the parameter b in (15) approaches 1 as the electron energy approaches its maximum $W_0 - m_\nu^2/2m_\mu$, the relative sensitivity of μ decay to a and m_ν is different from that of processes (3) and (4). To take the example of the last paragraph but one, if $a=0$ and $m_\nu=1.6$ MeV, the parameter b would affect both the energy spectrum and the asymmetry, for electron energies near the maximum, by an amount that might be observed by an improvement of existing measurements. But if $a=0.02$ and $m_\nu=0$, the parameter a^2 in (15) would be quite negligible. Thus, an improved measurement of the tail of the electron spectrum in μ decay might be combined with a measurement of the branching ratios for (3) and (4) to give an estimate of both a and m_ν.

The preceding considerations are theoretical. In the immediate future, however, there is little hope of observing reactions (3) and (4) unless the branching ratio R_3/R_1 is at the least 0.01. Such a large effect could not be produced by m_ν alone and could only be interpreted as evidence that $a^2>0$. Indeed, it is known from the energy-momentum balance in π-μ decay[6] that $m_\nu \lesssim 3\frac{1}{2}$ MeV. From Eq. (13) it follows that if $a=0$, R_3/R_1 is at most about 1/200.

An upper limit on a^2 may be obtained from the isotropic energy spectrum of the electrons in μ decay. If we neglect m_ν, the ρ value from Eq. (15) is $\frac{3}{4}(1+a^2)^{-1}$. Plano's[7] experimental value is 0.780 ± 0.025. If this is taken literally it is too high even if $a^2=0$. Let us therefore allow two standard deviations; then $\rho \geqslant 0.730$, $a^2 \leqslant 0.027$, and $R_3/R_1 \lesssim 5\frac{1}{2}/100$.

The maximum conceivable value of R_3/R_1 is obtained by letting a be positive and giving m_ν and a^2 their maximum values. Let us neglect the influence of $m_\nu \approx 3\frac{1}{2}$ MeV on the ρ value in μ decay, so that we can retain our maximum of 0.027 for a^2. Then the maximum value for R_3/R_1 is about 8/100.

ACKNOWLEDGMENTS

I am indebted to Professor T. D. Lee for much helpful advice with this problem. I would also like to thank Dr. Robert Oppenheimer for the hospitality of the Institute for Advanced Study, and the National Science Foundation for financial support.

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PHYSICAL REVIEW VOLUME 129, NUMBER 5 1 MARCH 1963

π^--p Elastic Scattering at 310 MeV: Differential Cross Section and Recoil-Proton Polarization*

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(Received 4 October 1962)

The differential cross section and recoil-proton polarization in π^--p elastic scattering at 310-MeV incident-pion energy has been measured. The differential cross section was measured at 28 angles in the angular region $25 \leqslant \theta_{\text{lab}} \leqslant 160$ deg. The fractional rms errors were typically 3%. The reaction was observed by counting the scattered pions emerging from a liquid-hydrogen target with a counter telescope consisting of scintillation and Čerenkov counters. Simultaneously, the recoil-proton polarization was measured at four angles in the angular region $114 < \theta_{\text{c.m.}} < 146$ deg. The recoil protons from the liquid-hydrogen target were scattered from a carbon target and the left-right asymmetry was measured. Scintillation counters were used throughout to detect the particles.

I. INTRODUCTION

THE pion-nucleon interaction has been experimentally studied for some time. In the majority of the experiments performed, differential and total cross sections for the elastic scattering of positive and negative pions on protons have been measured. Although there exists a reasonable amount of data, most of it lacks completeness and accuracy at any single energy, especially in the region near 300 MeV. The experiments described herein are a part of a continuing effort at the Lawrence Radiation Laboratory to obtain accurate and complete data on the π-N interaction at 310 MeV. Previously Rogers *et al.*[1] had made accurate measurements of the differential cross section for π^+-p elastic scattering, and Foote *et al.*[2] had obtained recoil-proton polarization for the same reaction. However, investigation of π^+-p scattering leads only to information about the isospin 3/2 state of the π-N system. To

* This work was done under the auspices of the U. S. Atomic Energy Commission.

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